MATHEMATICAL PHYSICS

MAT/07 - 12 CFU - 2° Semester

Teaching Staff

MASSIMO TROVATO
Email: trovato@dmi.unict.it
Office: Dipartimento di Matematica e Informatica, Blocco III, ufficio M37, Viale A. Doria 6, 95125 Catania
Phone: 0957383037
Office Hours: Mercoledi e Venerdi dalle ore 16:30 alle ore 19

LEARNING OBJECTIVES

The course has as main objective the theoretical treatment of classical mechanics allowing the student to connect the topics of the course with the concepts learned in Calculus I, Calculus II, Geometry I and General Physics I.

With this course the student will acquire the basic knowledge for:
i) The study of holonomic systems with particular regard to the kinematics and dynamics of rigid bodies.
ii) The Analytical Mechanics.

In particular, the course will aim to achieve the following objectives:

1) Knowledge and understanding:
One objective of the course shall be provide mathematic instruments, such as theorems and algoritms, which permit to face real problems in applied mathematics, physics, informatics, chemistry, economy and many other fields. With these mathematical instruments, student gets new abilities to clear useful theoretical and application problems.

2) Applying knowledge and understanding:
At the end of course student will be able to get new mathematical techniques of knowledge and understanding to face all possible links moreover, if it is possible, they will propose untreated new problems.

3) Making judgements:
Course is based on logical-deductive method which wants to give to students authonomus judgement useful to understanding incorrect method of demonstration also, by logical reasoning, student will be able to face not difficult problems, in applied mathematics, with teacher's help.

4) Communication skills:
In the final exam, student must show, for learned different mathematical techniques, an adapt maturity

on oral communication.

5) Learning skills:
Students must acquire the skills necessary to undertake further studies (master's degree) with a high degree of autonomy. The course in addition to proposing theoretical arguments presents arguments which also should be useful in different working fields.

DETAILED COURSE CONTENT

Vectorial and tensorial algebra:
Vector spaces, dimensions and bases of a vector space. Pseudo-Euclidean and Euclidean spaces. Metric tensor, covariant and contravariant components. Coordinates Cartesian, polar, spherical and Cylindrical. Coordinate changes. Curvilinear coordinates. calculus scalar and vector products, mixed products. Tensorial Algebra. Covariant, contravariant and mixed components of a tensor. Vector fields in physics

Kinematics:
Particle kinematics. Motion, velocity and acceleration of a point particle: plane, circular, harmonic and helical motions. Curvilinear abscissa. Intrinsic systems of references. Frénet formulas. Kinematics of rigid bodies. Poisson's formulas and angular velocity. Analysis of the field of velocity of a rigid body. Different kinds of rigid motions. Plane rigid motions. Rigid body with a fixed point. Rigid body with a fixed axis. Mechanics of rigid bodies, some applications. Relative kinematics. Composition of the velocities, of the accelerations and of the angular velocities. Galileian equivalence. Inertial frames and Galilei transformations. Inertial and not inertial systems of references. Coriolis theorem. Fictitious forces. Coriolis forces. Euler angles.

Dynamics:
Axioms of classical dynamics. Statics and the dynamics of a particle. Statics and the dynamics of a system. Cardinal equations in static and in dynamics. Conservation theorems. Rigid-body dynamics. Centers of mass and moments of inertia. Inertia tensor, principal axes. Principal moments of inertia. Properties of the inertia tensor. Huygens and Steiner's theorems. Koenig's theorem for the kinetic energy. Kinetic energy and angular momentum of a rigid body. Potential energy. Constraints. Holonomic and non-Holonomic constraints for physical systems. Generalized coordinates and degrees of freedom. Configuration space. Bilateral and unilateral constraints. Reversible and irreversible displacements. Ideal constraints. Possible and virtual displacements. Principle of virtual work. Principle of d'Alembert. Lagrangian and Lagrange equations. Conservative force fields and potentials. Conservation of energy. Generalized potentials and applications. Integrals of motion. Equilibrium positions. Stability of equilibrium positions. Lyapunov theorem. Dirichlet Stability Theorem. Small oscillations around stable equilibrium points.

Analytical Mechanics:
Variational principles and the Lagrange equations. Configuration space. Tangent vectors and tangent space. Variational principle and Hamilton principle in the Configuration space. Principle of the least action and the Lagrange equations. Cyclic variables. Geodetic calculations. The brachistochrone problem. Conserved quantities and Noether theorem. Two-body problem. Phase space. Hamiltonian Formalism. Legendre transformations. Hamilton equations. Derivation of Hamilton equations from a Variational principle. Application of Hamiltonian methods to various problems. Canonical transformations. Generating function of a canonical transformation. Applications and examples. The theory of Hamilton-Jacobi. Derivation of Hamilton-Jacobi equation from a Variational principle. Equation of Hamilton-Jacobi and its

application. Variable separation in the method of Hamilton-Jacobi. Poisson brackets. Poisson theorem. Applications and examples.

Variational principles in the theory of electromagnetic fields:

Introduction to special relativity, Lorentz transformations, time dilation and length contraction, world-lines, Minkowski metric, spacetime interval, 4-scalars, 4-vectors and 4-tensors. Lagrangian formulation and equations of motion deduced from variational principles.Variation of a functional in fields theory. Tensor of the Electromagnetic Field. Gauge invariance and its connection with potentials generalized. Invariants of the Electromagnetic Field. Construction of the Lagrangian function using the representation theorems for scalar functions of the Lorentz group. General formulation for the linear and nonlinear Maxwell equations, microscopic interpretation, experimental verification.

TEXTBOOK INFORMATION

1. Teacher's notes.
2. S. Rionero, Lezioni di Meccanica razionale, Liguori Editore.
3. Strumia Alberto, Meccanica razionale. Vol. 1 e Vol. 2, Ed. Nautilus Bologna (http://albertostrumia.it/?q=content/meccanica-razionale-parte-ii)
4. Strumia Alberto, Complementi di Meccanica Analitica (http://albertostrumia.it/?q=content/meccanica-razionale-parte-ii)
5. A.Fasano, V.De Rienzo, A.Messina, Corso di Meccanica Razionale, Laterza, Bari.
6. H. Goldstein, Meccanica classica, Zanichelli, Bologna.
7. L.D. Landau E. M. Lifsits, Fisica teorica. Vol. 1: Meccanica, Editori Riuniti.
8. Valter Moretti, Elementi di Meccanica Razionale, Meccanica Analitica e Teoria della Stabilità. (http://www.science.unitn.it/~moretti/runfismatI.pdf)
9. L.D. Landau E. M. Lifsits, Fisica teorica. Vol. 2: Teoria dei campi, Editori Riuniti.
